DAY ONE

Physical World and Measurement

Learning & Revision for the Day

- Physics
- Units
- Significant Figures
- Accuracy and Precision
- Errors in Measurement
- Dimensions of Physical Quantities

Physics

Physics is the study of matter and its motion, as well as space and time using concepts such as energy, force, mass and charge. It is an experimental science, creating theories that are tested against observation.

Scope and Excitement

Scope of Physics is very vast, as it deals with a wide variety of disciplines such as mechanics, heat, light, etc.

It also deals with very large magnitude of astronomical phenomenon as well as very small magnitude involving electrons, protons, etc.

Nature of Physical Laws

Physics is the study of nature and natural phenomena. All observations and experiments in physics lead to certain facts. These facts can be explained on the basis of certain laws.

Physics, Technology and Society

Connection between physics, technology and society can be seen in many examples like working of heat engines gave rise to thermodynamics. Wireless communication technology arose from basic laws of electricity and magnetism. Lately discovery of silicon chip triggered the computer revolution.

Units

Measurement of any physical quantity involves comparison with a certain basic, widely accepted reference standard called unit.

Fundamental and Derived Units

Fundamental units are the units which can neither be derived from one another, nor they can be further resolved into more simpler units.

These are the units of fundamental quantity. However, **derived units** are the units of measurement of all physical quantities which can be obtained from fundamental units.

System of Units

A complete set of these units, both fundamental and derived unit is known as the **system of units**.

The common systems are given below:

1. **CGS System** (Centimetre, Gram, Second) are often used in scientific work. This system measures, Length in centimetre (cm), Mass in gram (g), Time in second (s).
2. **FPS System** (Foot, Pound, Second) It is also called the British Unit System. This unit measures, Length in foot (foot), Mass in pound (pound), Time in second (s).
3. **MKS System** In this system also length, mass and time have been taken as fundamental quantities and corresponding fundamental units are metre, kilogram and second.
4. **International System (SI) of Units** It is an extended version of the MKS (Metre, Kilogram, Second) system. It has seven base units and two supplementary units. Seven base quantities and two supplementary quantities, their units along with definitions are tabulated below.

Base Quantity	Basic Units	
	Name and Symbol	Definintion
Length	metre (m)	The metre is the length of path travelled by light in vacuum during a time interval of 1/299, 792, 458 part of a second.
Mass	kilogram (kg)	It is the mass of the international prototype of the kilogram (a platinum iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres (France).
Time	second (s)	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of cesium-133 atom.
Electric current	Ampere (A)	The ampere is that constant current, which if maintained in two straight, parallel conductors of infinite length placed 1 m apart in vacuum would produce a force equal to 2×10^{-7} Nm^{-1} on either conductor.
Thermodynamic temperature	Kelvin (K)	The kelvin is $\frac{1}{273.16}$ th fraction of the thermodynamic temperature of the triple point of water.
Amount of substance	mole (mol)	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12.
Luminous intensity	candela (cd)	The candela is the luminous intensity in a given direction of a source emitting monochromatic radiation of frequency 540×10^{12} Hz and having a radiant intensity of $\frac{1}{683}$ W sr^{-1} in that direction.

Supplementary Quantity	Supplementary Units	
	Name and Symbol	Definition
Plane angle	radian (rad)	It is the angle subtended at the centre by an arc of a circle having a length equal to the radius of the circle.
Solid angle	steradian (sr)	It is the solid angle which is having its vertex at the centre of the sphere, it cuts-off an area of the surface of sphere equal to that of a square with the length of each side equal to the radius of the sphere.

NOTE
- Angle subtended by a closed curve at an inside points is 2π rad.
- Solid angle subtended by a closed surface at an inside point is 4π steradian.

Significant Figures

In the measured value of a physical quantity, the digits about the correction of which we are sure, plus the last digits which is doubtful, are called the significant figures.

Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement.

Accuracy and Precision

The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. However, precision tells us to what resolution or limit, the quantity is measured by a measuring instrument.

Least Count

The least count of a measuring instrument is the least value, that can be measured using the instrument. It is denoted as LC.

(i) **Least count of vernier callipers**

$$\text{LC} = \frac{\text{Value of 1 main scale division}}{\text{Total number of vernier scale division}}$$

(ii) **Least count of screw gauge**

$$\text{LC} = \frac{\text{Value of 1 pitch scale reading}}{\text{Total number of head scale division}}$$

Errors in Measurement

The difference in the true value (mean value) and measured value of a quantity is called error of measurement. Different types of error are given below:

(i) Absolute error,

$$a_{\text{mean}} = a_0 = \frac{a_1 + a_2 + a_3 + \dots a_n}{n} = \frac{1}{n}\sum_{i=1}^{i=n} a_i$$

$$\Delta a_1 = \text{mean value} - \text{observed value}$$

$$\Delta a_1 = a_0 - a_1$$

$$\Delta a_2 = a_0 - a_2$$

$$\vdots \quad \vdots \quad \vdots$$

$$\Delta a_n = a_0 - a_n$$

(ii) Mean absolute error,

$$\Delta a_{\text{mean}} = \frac{[|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|]}{n}$$

$$= \frac{\sum_{i=1}^{n} |\Delta a_i|}{n}$$

(iii) Relative or fractional error $= \dfrac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$

(v) Percentage error,

$$\delta_a = \text{Relative error} \times 100\,\% = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$$

Combination of Errors

(i) If $X = A + B$, then $(\Delta X) = \pm(\Delta A + \Delta B)$

(ii) If $X = ABC$, then $\left(\dfrac{\Delta X}{X}\right)_{\max} = \pm\left[\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B} + \dfrac{\Delta C}{C}\right]$

(iii) If $X = A^k B^l C^n$, then $\left(\dfrac{\Delta X}{X}\right) = \pm\left[k\dfrac{\Delta A}{A} + l\dfrac{\Delta B}{B} + n\dfrac{\Delta C}{C}\right]$

Dimensions of Physical Quantities

The dimensions of a physical quantity are the powers to which the fundamental (base) quantities are raised, to represent that quantity.

To make it clear, consider the physical quantity force.

'Force = mass × acceleration = mass × length × $(\text{time})^{-2}$'

Thus, the dimension of force are 1 in mass [M]

1 in length [L] and −2 in time $[\text{T}^{-2}]$, that is $[\text{MLT}^{-2}]$.

NOTE
- Dimensions of a physical quantity do not depend on its magnitude or the units in which it is measured.

Principle of Homogeneity of Dimensions and Applications

According to this principle, a correct dimensional equation must be homogeneous, i.e. dimensions of all the terms in a physical expression must be same.

$$\text{LHS} = \text{RHS}$$

Uses of Dimensions

(i) To check the correctness of a given physical equation.

(ii) Derivation of formula.

(iii) Dimensional formula is useful to convert the value of a physical quantity from one system to the other. Physical quantity is expressed as a product of numerical value and unit. In any system of measurement, this product remains constant.

Let dimensional formula of a given physical quantity be $[\text{M}^a \text{L}^b \text{T}^c]$. If in a system having base units $[\text{M}_1 \text{L}_1 \text{T}_1]$ the numerical value of given quantity be n_1 and numerical value n_2 in another unit system having the base units $[\text{M}_2, \text{L}_2, \text{T}_2]$, then $Q = n_1 u_1 = n_2 u_2$

$$n_1[\text{M}_1^a \text{L}_1^b \text{T}_1^c] = n_2[\text{M}_2^b \text{L}_2^b \text{T}_2^c]$$

$$n_2 = n_1\left[\frac{\text{M}_1}{\text{M}_2}\right]^a \left[\frac{\text{L}_1}{\text{L}_2}\right]^b \left[\frac{\text{T}_1}{\text{T}_2}\right]^c$$

Dimensions of Important Physical Quantities

Physical Quantity	SI Unit	Dimensional Formula
Power	Watt (W)	$[\text{ML}^2\text{T}^{-3}]$
Pressure, stress, coefficient of elasticity (ρ, σ, η)	Pascal (Pa) or Nm^{-2}	$[\text{ML}^{-1}\text{T}^{-2}]$
Frequency, angular frequency	Hz or s^{-1}	$[\text{T}^{-1}]$
Angular momentum	$\text{kg m}^2\text{s}^{-1}$	$[\text{ML}^2\text{T}^{-1}]$
Torque	Nm	$[\text{ML}^2\text{T}^{-2}]$
Gravitational constant (G)	$\text{N m}^2\text{kg}^{-2}$	$[\text{M}^{-1}\text{L}^3\text{T}^{-2}]$
Moment of inertia	kg m^2	$[\text{ML}^2]$
Acceleration, acceleration due to gravity	ms^{-2}	$[\text{LT}^{-2}]$
Force, thrust, tension, weight	Newton (N)	$[\text{MLT}^{-2}]$
Linear momentum, impulse	kg ms^{-1} or Ns	$[\text{MLT}^{-1}]$
Work, energy, KE, PE, thermal energy, internal energy, etc.	Joule (J)	$[\text{ML}^2\text{T}^{-2}]$
Surface area, area of cross-section	m^2	$[\text{L}^2]$
Electric conductivity	Sm^{-1}	$[\text{M}^{-1}\text{L}^{-3}\text{T}^3\text{A}^2]$
Young's modulus, Bulk modulus	Pa	$[\text{ML}^{-1}\text{T}^{-2}]$
Compressibility	$\text{m}^2\ \text{N}^{-1}$	$[\text{M}^{-1}\text{LT}^2]$
Magnetic Flux	Wb	$[\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$
Magnetic Flux density (σ)	Wb/m^2	$[\text{MT}^{-2}\text{A}^{-1}]$
Intensity of a wave	Wm^{-2}	$[\text{MT}^{-3}]$
Photon flux density	$\text{m}^{-2}\ \text{s}^{-1}$	$[\text{L}^{-2}\text{T}^{-1}]$
Luminous energy	Lm s	$[\text{ML}^2\text{T}^{-2}]$
Luminance	Lux	$[\text{MT}^{-3}]$

Physical Quantity	SI Unit	Dimensional Formula
Specific heat capacity	$\text{Jkg}^{-1}\text{K}^{-1}$	$[\text{L}^2\ \text{T}^{-2}\ \text{K}^{-1}]$
Latent heat of vaporisation	Jkg^{-1}	$[\text{L}^2\text{T}^{-2}]$
Coefficient of Thermal conductivity	$\text{Wm}^{-1}\text{K}^{-1}$	$[\text{MLT}^{-3}\ \text{K}^{-1}]$
Electric voltage	JC^{-1}	$[\text{ML}^2\text{T}^{-3}\text{A}^{-1}]$

Physical Quantity	SI Unit	Dimensional Formula
Magnetisation	Am^{-1}	$[\text{L}^{-1}\text{A}]$
Magnetic induction	T	$[\text{MT}^{-2}\text{A}^{-1}]$
Planck's constant	J-s	$[\text{ML}^2\text{T}^{-1}]$
Radioactive decay constant	Bq	$[\text{T}^{-1}]$
Binding energy	MeV	$[\text{ML}^2\text{T}^{-2}]$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The numerical value of a given quantity is

(a) independent of unit
(b) directly proportional to unit
(c) inversely proportional to unit
(d) directly proportional to the square root of the unit

2 Unit of reduction factor is

(a) ampere (b) ohm (c) tesla (d) weber

3 Lumen is the unit of

(a) illuminating power (b) luminous flux
(c) luminous intensity (d) None of these

4 Which one of the following is not a unit of Young's modulus?

(a) Nm^{-1} (b) Nm^{-2}
(c) Dyne cm^{-2} (d) Mega pascal

5 Young's modulus of the material of a wire is 18×10^{11} dyne cm^{-2}. Its value in SI is

(a) $18 \times 10^{15}\ \text{Nm}^{-2}$ (b) $18 \times 10^{10}\ \text{Nm}^{-2}$
(c) $18 \times 10^{9}\ \text{Nm}^{-2}$ (d) $18 \times 10^{12}\ \text{Nm}^{-2}$

6 Which of the following measurement is most precise?

(a) 5.00 mm 1(b) 5.00 cm
(c) 5.00 m (d) 5.00 km

7 The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 5, 1, 2 (b) 5, 1, 5 (c) 5, 5, 2 (d) 4, 4, 2

8 A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of –0.004 cm, the correct diameter of the ball is ➜ NEET 2018

(a) 0.053 cm (b) 0.525 cm (c) 0.521 cm (d) 0.529 cm

9 If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be

(a) 4% (b) 6% (c) 8% (d) 2%

10 The density of a cube is measured by measuring its mass and length of its sides. If the maximum error in the measurement of mass and length are 4% and 3%, respectively, the maximum error in the measurement of density will be

(a) 7% (b) 9% (c) 12% (d) 13%

11 In an experiment four quantities *a*, *b*, *c* and *d* are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity *P* is calculated as follows

$P = \dfrac{a^3 b^2}{cd}$%. Error in *P* is ➜ NEET 2013

(a) 14% (b) 10%
(c) 7% (d) 4%

12 If force (F), length (L) and time (T) be considered fundamental units, then the units of mass will be

(a) $[\text{FLT}^{-2}]$ (b) $[\text{FL}^{-2}\text{T}^{-1}]$
(c) $[\text{FL}^{-1}\text{T}^{2}]$ (d) $[\text{F}^2\text{LT}^{-2}]$

13 Which of the following quantities has units but not dimensions?

(a) Displacement (b) Angle
(c) Couple (d) Speed

14 Pascal-second has the dimensions of

(a) force (b) energy
(c) pressure (d) coefficient of viscosity

15 The physical quantity which does not have the same dimensions as the other three is

(a) spring constant
(b) surface tension
(c) surface energy
(d) acceleration due to gravity

16 Pressure gradient has the same dimensions as that of

(a) velocity gradient
(b) potential gradient
(c) energy gradient
(d) None of these

17 Dimensions of the resistance in an electrical circuit in terms of dimension of mass M, length L, time T and current I, are

(a) $[ML^2T^{-3}A^{-1}]$ (b) $[ML^2T^{-2}]$
(c) $[ML^2T^{-1}A^{-1}]$ (d) $[ML^2T^{-3}A^{-2}]$

18 The dimensions of $(\mu_0\varepsilon_0)^{-\frac{1}{2}}$ are ➔ CBSE-AIPMT 2011

(a) $[L^{-1}T]$ (b) $[LT^{-1}]$ (c) $[L^{-1/2}\,T^{1/2}]$ (d) $[L^{1/2}T^{-1/2}]$

19 If L is the inductance, C capacitance and R resistance the ratios L/R and R-C have the same dimensions as those of

(a) frequency (b) time (c) energy (d) length

20 The ratio of the dimensions of Planck's constant and that of the moment of inertia is the dimensions of

(a) frequency (b) velocity
(b) angular momentum (d) time

21 If energy (E), velocity (v) and time (T) are chosen as the fundamental quantities, the dimensional formula of surface tension will be ➔ CBSE AIPMT 2015

(a) $[Ev^{-1}T^{-2}]$ (b) $[Ev^{-2}T^{-2}]$
(c) $[E^{-2}v^{-1}T^{-3}]$ (d) $[Ev^{-2}T^{-1}]$

22 If force (F), velocity (v) and time (T) are taken as fundamental units, then the dimensions of mass are ➔ CBSE AIPMT 2014

(a) $[FvT^{-1}]$ (b) $[FvT^{-2}]$ (c) $[Fv^{-1}T^{-1}]$ (d) $[Fv^{-1}T]$

23 The dimensions of $\frac{1}{2}\varepsilon_0 E^2$, where ε_0 is permittivity of free space and E is electric field, are ➔ CBSE AIPMT 2010

(a) $[ML^2T^{-2}]$ (b) $[ML^{-1}T^{-2}]$ (c) $[ML^2T^{-1}]$ (d) $[MLT^{-1}]$

24 Velocity of sound in a gas is given by $v = \sqrt{\frac{\gamma p}{\rho}}$.

Dimensional formula for γ is

(a) $[MLT]$ (b) $[M^0L^0T^0]$ (c) $[M^0LT^0]$ (d) $[ML^0T^0]$

25 In the equation $y = a\sin(\omega t + kx)$, the dimensional formula of ω is

(a) $[M^0L^0T^{-1}]$ (b) $[M^0LT^{-1}]$
(c) $[ML^0T^0]$ (d) $[M^0L^{-1}T^0]$

26 In the relation $p = \frac{\alpha}{\beta}e^{\frac{-\alpha z}{k\theta}}$, p is pressure, z is distance, k is Boltzmann constant and θ is temperature. The dimensional formula of β will be

(a) $[M^0L^2T^0]$ (b) $[ML^2T]$
(c) $[ML^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

27 If momentum (p), area (A) and time (T) are taken to be fundamental quantities, the energy has the dimensional formula

(a) $[pA^{-1}T^1]$ (b) $[p^2A^1T^1]$ (c) $[p^1A^{-1/2}T^1]$ (d) $[p^1A^{1/2}T^{-1}]$

28 The refractive index of a material is given by the equation $n = A + \frac{B}{\lambda^2}$, where A and B are constants. The dimensional formula for B is

(a) $[M^0L^2T]$ (b) $[M^0L^{-2}T^0]$
(c) $[M^0L^2T^{-2}]$ (d) $[M^0L^2T^0]$

29 If the dimensions of a physical quantity are given by $[M^aL^bT^c]$, then the physical quantity will be ➔ CBSE AIPMT 2009

(a) pressure if $a = 1$, $b = -1$, $c = -2$
(b) velocity if $a = 1$, $b = 0$, $c = -1$
(c) acceleration if $a = 1$, $b = 1$, $c = -2$
(d) force if $a = 0$, $b = -1$, $c = -2$

30 If $F = 6\pi\eta^a r^b v^c$, where, F = viscous force,
η = coefficient of viscosity, r = radius of spherical body,
v = terminal velocity of the body

The values of a, b and c are

(a) $a = 1$, $b = 2$, $c = 1$ (b) $a = 1$, $b = 1$, $c = 1$
(c) $a = 2$, $b = 1$, $c = 1$ (d) $a = 2$, $b = 1$, $c = 2$

31 If dimensions of critical velocity v_c of a liquid flowing through a tube are expressed as $[\eta^x\rho^y r^z]$, where η, ρ and r are the coefficient of viscosity of liquid, density of liquid and radius of the tube respectively, then the value of x, y and z are given by ➔ CBSE AIPMT 2015

(a) 1, – 1, – 1 (b) –1, – 1, 1
(c) –1, – 1, – 1 (d) 1, 1, 1

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 1 Wb/ m^2 is equal to .

(a) 10^4 G (b) 10^2 G
(c) 10^{-2} G (d) 10^{-4} G

2 The magnetic moment has dimensions of

(a) $[L\,A]$ (b) $[L^2A]$
(c) $[LT^{-1}A]$ (d) $[L^2T^{-1}A]$

3 SI unit of permittivity is

(a) $C^2m^2N^2$ (b) $C^2m^{-2}N^{-1}$
(c) $C^2m^2N^{-1}$ (d) $C^{-1}m^2N^2$

4 If h is Planck's constant and λ is wavelength, h/λ has dimensions of

(a) momentum (b) energy (c) mass (d) velocity

5 The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is

(a) $(164 \pm 3)\ \text{cm}^2$ (b) $(163.62 \pm 2.6)\ \text{cm}^2$
(c) $(163.6 \pm 2.6)\ \text{cm}^2$ (d) $(163.62 \pm 3)\ \text{cm}^2$

6 Which of the following pairs of physical quantities does not have same dimensional formulae?

(a) Work and torque
(b) Angular momentum and planck's constant
(c) Tension and surface tension
(d) Impulse and linear momentum

7. If E, M, L and G denote energy, mass, angular momentum and gravitational constant respectively, then the quantity (E^2L^2/M^5G^2) has the dimensions of

(a) angle (b) length
(c) mass (d) None of these

8 van der Waals', equation of state is $\left(p + \frac{a}{V^2}\right)(V - b) = nRT$. The dimensions of a and b are

(a) $[\text{ML}^3\text{T}^2]$, $[\text{ML}^3\text{T}^0]$ (b) $[\text{ML}^5\text{T}^{-2}]$, $[\text{M}^0\text{L}^3\text{T}^0]$
(c) $[\text{M}^2\text{LT}^2]$, $[\text{ML}^3\text{T}^2]$ (d) $[\text{ML}^2\text{T}]$, $[\text{ML}^2\text{T}^2]$

9 According to Newton, the viscous force acting between liquid layers of area A and velocity gradient $\frac{\Delta v}{\Delta z}$ is given by $F = -\eta A \frac{dv}{dz}$, where η is constant called coefficient of viscosity. The dimensional formula of η is

(a) $[\text{ML}^{-2}\text{T}^{-2}]$ (b) $[\text{M}^0\text{L}^0\text{T}^0]$
(c) $[\text{ML}^2\text{T}^{-2}]$ (d) $[\text{ML}^{-1}\text{T}^{-1}]$

10 A physical quantity is given by $X = [\text{M}^a\text{L}^b\text{T}^c]$. The percentage error in measurements of M, L and T are α, β and γ. Then, the maximum % error in the quantity X is

(a) $a\alpha + b\beta + c\gamma$ (b) $a\alpha + b\beta - c\gamma$
(c) $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}$ (d) None of these

11 The frequency of vibration f of a mass m suspended from a spring of spring constant k is given by a relation of the type $f = Cm^xk^y$, where C is a dimensionless constant. The values of x and y are

(a) $x = \frac{1}{2}, y = \frac{1}{2}$ (b) $x = -\frac{1}{2}, y = -\frac{1}{2}$
(c) $x = \frac{1}{2}, y = -\frac{1}{2}$ (d) $x = -\frac{1}{2}, y = \frac{1}{2}$

12 A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(a) A meter scale
(b) A Vernier calliper, where the 10 divisions in Vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

13 Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is

(a) 6% (b) zero (c) 1% (d) 3%

14 A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and e is charge]

➡ NEET 2017

(a) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ (b) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$
(c) $\frac{1}{c^2}\left[\frac{e^2}{G\,4\pi\varepsilon_0}\right]^{1/2}$ (d) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$

15. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20.0 cm known to 1mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of resolution. The accuracy in the determination of g is

(a) 2% (b) 3% (c) 1% (d) 5%

ANSWERS

SESSION 1	**1** (c)	**2** (a)	**3** (b)	**4** (a)	**5** (b)	**6** (a)	**7** (a)	**8** (d)	**9** (b)	**10** (d)
	11 (a)	**12** (c)	**13** (b)	**14** (d)	**15** (d)	**16** (d)	**17** (d)	**18** (b)	**19** (b)	**20** (a)
	21 (b)	**22** (d)	**23** (b)	**24** (b)	**25** (a)	**26** (a)	**27** (d)	**28** (d)	**29** (a)	**30** (b)
	31 (a)									
SESSION 2	**1** (a)	**2** (b)	**3** (b)	**4** (a)	**5** (a)	**6** (c)	**7** (d)	**8** (b)	**9** (d)	**10** (a)
	11 (d)	**12** (b)	**13** (a)	**14** (a)	**15** (b)					

Hints and Explanations

SESSION 1

1 In general, $n[u] = \text{constant}$

where, n = numerical value,

u = unit of physical quantity

$$n \propto \frac{1}{[u]}$$

2 Current flowing in the coil of tangent galvanometer is given by $I = K\phi$

where, K is a constant called reduction factor and ϕ is the angle of deflection. Since, deflection has no unit. So, unit of reduction factor is same as of current, i.e. ampere.

3 The lumen (lm) is the SI derived unit of luminous flux, a meausre of total quantity of visible light emitted by a source.

4 Young's modulus,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{N}}{\text{m}^2} \text{ or pascal} \qquad \text{[in SI system]}$$

$$Y = \frac{\text{dyne}}{\text{cm}^2} \qquad \text{[in CGS system]}$$

Hence, Nm^{-1} is not the unit of Young's modulus.

5 Unit of Young's modulus Y in SI units is Nm^{-2}.

Here, $Y = 18 \times 10^{11} \dfrac{\text{dyne}}{\text{cm}^2}$

As, $1 \text{ dyne} = 10^{-5} \text{ N}$

$1 \text{ cm} = 10^{-2} \text{ m}$

$$\Rightarrow \quad Y = 18 \times 10^{11} \times \frac{10^{-5}}{(10^{-2})^2} \text{ Nm}^{-2}$$

$$= 18 \times 10^{10} \text{ Nm}^{-2}$$

6 All measurements are correct upto two places of decimal. However, the absolute error in (a) is 0.01 mm, which is the least of all the four. So, 5.00 mm is most precise.

7 Number of significant figures in $23.023 = 5$

Number of significant figures in $0.0003 = 1$

Number of significant figures in $2.1 \times 10^{-3} = 2$

8 Given, least count of screw gauge,

$$\text{LC} = 0.001 \text{ cm}$$

Main scale reading,

$$\text{MSR} = 5 \text{ mm} = 0.5 \text{cm}$$

Number of coinciding divisions on the circular scale, i.e. Vernier scale reading, VSR = 25

Here, zero error = −0.004 cm

Final reading obtained from the screw gauge is given as

$$= \text{MSR} + \text{VSR} \times \text{LC} - \text{zero error}$$

Final reading from the screw gauge

$$= 0.5 + 25 \times 0.001 - (-0.004)$$

$$= 0.5 + 0.025 + 0.004$$

$$= 0.5 + 0.029 = 0.529 \text{ cm}$$

Thus, the diameter of the ball is 0.529 cm.

9 Volume of a sphere, $V = \dfrac{4}{3} \pi r^3$

$$\therefore \quad \frac{\Delta V}{V} \times 100 = \frac{3 \times \Delta r}{r} \times 100$$

$$\text{Here} \quad \frac{\Delta r}{r} \times 100 = 2\%$$

$$\therefore \quad \frac{\Delta V}{V} \times 100 = 3 \times 2\% = 6\%$$

10 Density, $\rho = \dfrac{m}{v} = \dfrac{m}{l^3}$

$$\Rightarrow \frac{\Delta \rho}{\rho} \times 100 = \pm \left(\frac{\Delta m}{m} + 3 \frac{\Delta l}{l} \right) \times 100$$

$$= \pm (4 + 3 \times 3)\% = \pm 13\%$$

11 Here, $P = \dfrac{a^3 b^2}{cd}$

$$\therefore \quad \frac{\Delta P}{P} \times 100$$

$$= \left(\frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d} \right) \times 100$$

$$= 3 \frac{\Delta a}{a} \times 100 + 2 \frac{\Delta b}{b} \times 100 + \frac{\Delta c}{c} \times 100 + \frac{\Delta d}{d} \times 100$$

$$= 3 \times 1 + 2 \times 2 + 3 + 4$$

$$= 3 + 4 + 3 + 4 = 14\%$$

12 Let $\quad M \propto [\text{F}^a \text{L}^b \text{T}^c]$

Writing dimensions of both sides and using the principle of homogeneity of dimensions, we have

$$[\text{M}^1 \text{L}^0 \text{T}^0] = K [\text{MLT}^{-2}]^a [\text{L}]^b [\text{T}]^c$$

On comparing the powers both sides

$$a = 1, \ a + b = 0, \ -2a + c = 0$$

On solving, we have

$$b = -1, \ c = 2, \ a = 1$$

$\therefore$ Units of mass is $[\text{FL}^{-1} \text{T}^2]$.

13 As we know that,

$$\text{Angle} = \frac{\text{arc}}{\text{radius}} = \frac{[L]}{[L]} = \text{dimensionless}$$

But unit of angle is radian.

14 Pascal is unit of pressure, hence its dimensional formula is $[\text{ML}^{-1} \text{T}^{-2}]$.

$\therefore$ Dimensional formula of Pascal-second is $[\text{ML}^{-1} \text{T}^{-1}]$.

From the formula of coefficient of viscosity, we have

$$\eta = \frac{F}{A(\Delta v / \Delta z)}$$

where, F is force, A is area and $\dfrac{\Delta v}{\Delta z}$ is velocity gradient.

$$\therefore \quad \text{Dimensions of } \eta = \frac{[\text{MLT}^{-2}]}{[\text{L}^2] [\text{LT}^{-1} / \text{L}]}$$

$$= [\text{ML}^{-1} \text{T}^{-1}]$$

Hence, Pascal-second has dimensions of coefficient of viscosity.

15 Spring constant, surface tension and surface energy have the same dimensions, i.e. $[\text{ML}^0 \text{T}^{-2}]$. However, acceleration due to gravity has dimensions $[\text{LT}^{-2}]$.

16 Pressure gradient $= \dfrac{\text{kg-m}^{-1}/\text{s}^2}{\text{m}} = [\text{ML}^{-2} \text{T}^{-2}]$

Velocity gradient $= \dfrac{\text{m/s}}{\text{m}} = [\text{M}^0 \text{L}^0 \text{T}^{-1}]$

Potential gradient $= \dfrac{\text{kg-m}^2/\text{s}^3\text{-A}}{\text{m}}$

$$= [\text{MLT}^{-3} \text{A}^{-1}]$$

Energy gradient $= \dfrac{\text{kg-m}^2/\text{s}^2}{\text{m}} = [\text{MLT}^{-2}]$

Hence option (d) is correct.

17 Resistance,

$$R = \frac{\text{Potential difference}}{\text{Current}} = \frac{V}{I} = \frac{W}{QI}$$

$$\left[\because V = \frac{W}{Q} \right]$$

So, dimension of R

$$= \frac{\text{[Dimension of work]}}{\text{[Dimension of charge]} \ \text{[Dimension of current]}} = \frac{[\text{ML}^2 \text{T}^{-2}]}{[\text{AT}] [\text{A}]}$$

$$= [\text{ML}^2 \text{T}^{-3} \text{A}^{-2}]$$

18 We know that, $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

where, c is speed of light.

Hence, dimensions of $(\mu_0 \varepsilon_0)^{-1/2}$ is equal to that speed of light $[\text{LT}^{-1}]$.

19 Dimensional formula of L is $[\text{ML}^2 \text{T}^{-2} \text{A}^{-2}]$, C is $[\text{M}^{-1} \text{L}^{-2} \text{T}^4 \text{A}^2]$ and that of R is $[\text{ML}^2 \text{T}^{-3} \text{A}^{-2}]$. Thus, dimensional formula of L/R is $[\text{T}]$.

Similarly, dimensional formula of R-C is $[\text{T}]$.

$\therefore$ L/R and R-C has the dimensions of time.

20 From Einstein's equation, $E = h\nu$

$$\Rightarrow \quad h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]}$$

Dimensions of Planck's constant,

$$[h] = [ML^2T^{-1}]$$

Also, moment of inertia, $I = MR^2$

$$\Rightarrow \quad [I] = [ML^2]$$

$$\therefore \quad \frac{[h]}{[I]} = \frac{[ML^2T^{-1}]}{[ML^2]} = [T^{-1}]$$

= dimension of frequency.

21 Surface tension

$$= \frac{\text{Force}}{\text{Length}} = \frac{\text{Surface energy}}{\text{Area}}$$

$$= \frac{[E]}{[v \cdot T]^2} = [Ev^{-2}T^{-2}]$$

22 We know that, $F = Ma$

$$\Rightarrow \quad F = \frac{Mv}{t} \text{ or } M = \frac{Ft}{v}$$

$\therefore$ Dimension of $M = [Fv^{-1}T]$

23 As we know that,

Dimension of $\varepsilon_0 = [M^{-1}L^{-3}T^4A^2]$

Dimension of $E = [MLT^{-3}A^{-1}]$

So, dimension of

$$\frac{1}{2}\varepsilon_0 E^2 = [M^{-1}L^{-3}T^4A^2] \times [MLT^{-3}A^{-1}]^2$$

$$= [ML^{-1}T^{-2}]$$

24 γ is the ratio of C_p to C_V which has no unit.

$\therefore$ Its dimensional formula is $[M^0L^0T^0]$.

25 Given, $y = a \sin(\omega t + kx)$

Here, ω = angular velocity $= \frac{v}{r}$

$$\therefore \quad [\omega] = \frac{[v]}{[r]} = \frac{[LT^{-1}]}{[L]} = [T^{-1}]$$

$$= [M^0L^0T^{-1}]$$

26 Using the principle of homogeneity of dimensions, the quantity $\frac{\alpha z}{k\theta}$ is dimensionless.

$$\Rightarrow \quad \alpha = \frac{k\theta}{z}$$

$$\Rightarrow [\alpha] = \frac{[ML^2T^{-2}K^{-1}] \times [K]}{[L]} = [MLT^{-2}]$$

Also, $p = \frac{\alpha}{\beta}$

$$\Rightarrow \quad [\beta] = \left[\frac{\alpha}{p}\right] = \frac{[MLT^{-2}]}{[ML^{-1}T^{-2}]} = [M^0L^2T^0]$$

27 Let energy $E \propto p^a A^b T^c$

$$= K p^a A^b T^c$$

where, K is dimensionless constant.

$$\Rightarrow [M^1L^2T^{-2}] = [MLT^{-1}]^a [L^2]^b T^c$$

$$= M^a L^{a+2b} T^{-a+c}$$

Applying principle of homogeneity, we get $a = 1$, $a + 2b = 2$ and $-a + c = -2$

Solving it, we get

$$a = 1, b = 1/2 \text{ and } c = -1$$

$$\therefore \quad E = K[p^1A^{1/2}T^{-1}]$$

28 Refractive index, $n = A + \frac{B}{\lambda^2}$

From principle of homogeneity of dimensions, quantity B/λ^2 should have the dimensions of n

$$\Rightarrow \quad \left[\frac{B}{\lambda^2}\right] = [M^0L^0T^0]$$

$$\therefore \quad B = [M^0L^0T^0] \times [L^2] = [M^0L^2T^0]$$

29 (i) Dimensions of pressure $= [M^1L^{-1}T^{-2}]$

$\therefore$ If $a = 1, b = -1, c = -2$, then the physical quantity is pressure.

(ii) Dimensions of velocity $= [M^0L^1T^{-1}]$

Here, $a = 0, b = 1, c = -1$

(iii) Dimensions of acceleration $= [M^0L^1T^{-2}]$

Here, $a = 0, b = 1, c = -2$

(iv) Dimensions of force $= [M^1L^1T^{-2}]$

Here, $a = 1, b = 1, c = -2$

30 According to homogeneity principle,

$$[F] = [6\pi\eta^a r^b v^c]$$

or $[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$

or $[MLT^{-2}] = [M^a L^{-a+b+c} T^{-a-c}]$

Equating the coefficients and powers, we get

$\therefore \quad a = 1, -a + b + c = 1$

and $\quad -a - c = -2$

After solving $a = 1, b = 1, c = 1$

31 Given critical velocity of liquid flowing through a tube as expressed as

$v_c \propto \eta^x \rho^y r^z$

Coefficient of viscocity of liquid,

$$\eta = [ML^{-1}T^{-1}]$$

Density of liquid, $\rho = [ML^{-3}]$

Radius of a tube $r = [L]$

Critical velocity of liquid

$$v_c = [M^0L^1T^{-1}]$$

$$\Rightarrow [M^0L^1T^{-1}] = [ML^{-1}T^{-1}]^x [ML^{-3}]^y [L]^z$$

$$[M^0L^1T^{-1}] = [M^{x+y}L^{-x-3y+z}T^{-x}]$$

Comparing exponents of M, L and T, we get

$$x + y = 0, -x - 3y + z = 1, -x = -1$$

$$\Rightarrow \quad z = -1, x = 1, y = -1$$

SESSION 2

1 We know that, $1\text{ G} = 10^{-4}\text{ T}$

$$\Rightarrow \quad 1\text{ T} = \frac{1}{10^{-4}}\text{G} = 10^4\text{G}$$

Also, $\quad 1\text{ T} = 1\text{ Wb/m}^2$

$$\Rightarrow \quad 1\text{ Wb/m}^2 = 10^4\text{G}$$

2 Magnetic moment, $M = IA$

Thus, dimensions of $M = [A][L^2] = [L^2A]$

3 Coulomb's law, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$$\Rightarrow \quad \varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

$$= \frac{C^2}{Nm^2}$$

$$= C^2m^{-2}N^{-1}$$

4 We know that, wavelength $\lambda = \frac{h}{mv}$

$$\therefore \quad \frac{h}{\lambda} = mv.$$

Hence, h/λ has dimensions of momentum.

5 Here, $l = (16.2 \pm 0.1)$ cm,

$$b = (10.1 \pm 0.1)\text{ cm}$$

$$A = l \times b = 16.2 \times 10.1 = 163.62$$

Rounding off to one significant figure,

$$A = 164\text{ cm}^2$$

$$\frac{\Delta A}{A} = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right)$$

$$= \frac{0.1}{16.2} + \frac{0.1}{10.1}$$

$$= \frac{1.01 + 1.62}{16.2 \times 10.1} = 2.63\text{ cm}^2$$

Rounding off to one significant figure,

$\Delta A = 3\text{cm}^2$

$$\therefore \quad A = (164 \pm 3)\text{cm}^2$$

6 Tension = Force $= [M^1L^1T^{-2}]$

Surface tension

$$= \frac{\text{Force}}{\text{Length}} = \frac{[M^1L^1T^{-2}]}{[L]}$$

$$= [M^1L^0T^{-2}]$$

So, tension and surface tension does not have same dimensional formulae.

7 The dimensions of $E = [ML^2T^{-2}]$

Dimensions of $M = [M]$

Dimensions of $L = [ML^2T^{-1}]$

Dimensions of $G = [M^{-1}L^3T^{-2}]$

$\therefore$ Dimensions of

$$\left[\frac{E^2L^2}{M^5G^2}\right] = \frac{[ML^2T^{-2}]^2 [ML^2T^{-1}]^2}{[M]^5 [M^{-1}L^3T^{-2}]^2}$$

$$= [ML^2T^{-2}]$$

8 van der Waals' equation,

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

a/V^2 should have dimensions of pressure.

$\therefore$ a = pressure $\times V^2$

$$= [ML^{-1}T^{-2}][L^3]^2 = [ML^5T^{-2}]$$

b should have dimensions of volume, i.e. $[M^0L^3T^0]$

9 As, $F = -\eta A \dfrac{dv}{dz}$

$\therefore \quad \eta = -\dfrac{F}{A\left(\dfrac{dv}{dz}\right)}$

As $F = [\mathrm{MLT^{-2}}]$, $A = [\mathrm{L^2}]$

$dv = [\mathrm{LT^{-1}}], dz = [\mathrm{L}]$

$\therefore \quad \eta = \dfrac{[\mathrm{MLT^{-2}}]\,[\mathrm{L}]}{[\mathrm{L^2}]\,[\mathrm{LT^{-1}}]}$

$= [\mathrm{ML^{-1}T^{-1}}]$

10 Maximum possible % error is

$= \dfrac{\Delta X}{X} \times 100$

$= a\dfrac{\Delta \mathrm{M}}{\mathrm{M}} + b\dfrac{\Delta \mathrm{L}}{\mathrm{L}} + c\dfrac{\Delta \mathrm{T}}{\mathrm{T}}$

$= a\alpha + b\beta + c\gamma$

11 As, $f = Cm^x k^y$

$\therefore$ (Dimension of f)

$= C$ (dimension of $m)^x$

$\times$ (dimensions of $k)^y$

$[\mathrm{T^{-1}}] = C\,[\mathrm{M}]^x\,[\mathrm{MT^{-2}}]^y$ ...(i)

Applying the principle of homogeneity of dimensions, we get

$x + y = 0, -2y = -1$

or $y = \dfrac{1}{2}$

$\therefore \quad x = -\dfrac{1}{2}$

12 If student measures 3.50 cm, it means that there is an uncertainly of order 0.01cm.

For Vernier scale with 1 MSD = 1 mm and 9 MSD = 10 VSD

LC of Vernier calliper = 1 MSD – 1VSD

$= \dfrac{1}{10}\left(1 - \dfrac{9}{10}\right) = \dfrac{1}{100}\,\mathrm{cm}$

13 From Ohm's law, $R = \dfrac{V}{I}$

$\Rightarrow \ln R = \ln V - \ln I$

$\Rightarrow \dfrac{\Delta R}{R} = \dfrac{\Delta V}{V} + \dfrac{\Delta I}{I}$

$= 3\% + 3\% = 6\%$

14 As force, $F = \dfrac{e^2}{4\pi\varepsilon_0 r^2} \Rightarrow \dfrac{e^2}{4\pi\varepsilon_0} = r^2 \cdot F$

Putting dimensions of r and F, we get,

$\Rightarrow \left[\dfrac{e^2}{4\pi\varepsilon_0}\right] = [\mathrm{ML^3T^{-2}}]$ ...(i)

Also, force, $F = \dfrac{Gm^2}{r^2}$

$\Rightarrow \quad [G] = \dfrac{[\mathrm{MLT^{-2}}][\mathrm{L^2}]}{[\mathrm{M^2}]}$

$\Rightarrow \quad [G] = [\mathrm{M^{-1}L^3T^{-2}}]$ ...(ii)

and $\left[\dfrac{1}{c^2}\right] = \dfrac{1}{[\mathrm{L^2T^{-2}}]} = [\mathrm{L^{-2}T^2}]$ ...(iii)

Using Eqs. (i), (ii) and (iii), we get

$\dfrac{1}{c^2}\left(\dfrac{Ge^2}{4\pi\varepsilon_0}\right)^{1/2}$

$= [\mathrm{L^{-2}T^2}][[\mathrm{ML^3T^{-2}}][\mathrm{M^{-1}L^3T^{-2}}]]^{1/2}$

$= [\mathrm{L^{-2}T^2}][\mathrm{L^3T^{-2}}]$

$= [\mathrm{L}]$

15 Given, time period, $T = 2\pi\sqrt{\dfrac{L}{g}}$

Thus,changes can be expressed as

$= \dfrac{2\Delta T}{T} = \pm\dfrac{\Delta L}{L} \pm \dfrac{\Delta g}{g}$

According to the question, we can write

$\dfrac{\Delta L}{L} = \dfrac{0.1\mathrm{cm}}{20.0\mathrm{cm}} = \dfrac{1}{200}$

Again, time period, $T = \dfrac{90}{100}\,\mathrm{s}$ and

$\Delta T = \dfrac{1}{200}\,\mathrm{s} \Rightarrow \dfrac{\Delta T}{T} = \dfrac{1}{90}$

$\therefore \quad \dfrac{\Delta g}{g} = \dfrac{\Delta L}{L} + \dfrac{2\Delta T}{T}$

or $\dfrac{\Delta g}{g} \times 100\% = \left(\dfrac{\Delta L}{L}\right) \times 100\%$

$+ \left(\dfrac{2\Delta T}{T}\right) \times 100\%$

$= \left(\dfrac{1}{200} \times 100\right)\% + 2 \times \dfrac{1}{90} \times 100\%$

$= 2.72\% = 3\%$

Thus, accuracy in the determination of g is approximately 3 %.